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Comparative study of the performance of the CuSum and EWMA control charts

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Abstract

This work presents a comparative study of the performance of the cumulative sum (CuSum), as well as the exponentially weighted moving average (EWMA) control charts. The objective of this research is to verify when CuSum and EWMA control charts do the best control region, in order to detect small changes in the process average. Starting from the data of a productive process, several series were simulated. CuSum and EWMA control charts were used to determine the average run length (ARL) to detect a condition out of control. ARL found by each chart which was then, compared. It was observed that the CuSum control chart practically did not sign points out of control for the levels of variation between ± 1.0 standard deviation. For these variation levels the EWMA control chart was more efficient than CuSum. Among the parameters EWMA control chart the ones with constant $\lambda = 0.10$ and 0.05, with the respective control limits $L = 2.814$ and 2.625, were the ones that detected larger number of altered positions.

Keywords: Cumulative sum; Exponentially weighted moving average; Average runs length; X-bar chart; Statistical process control

1. Introduction

The main purpose of Statistical Process Control is to improve the quality and productivity. One of the instruments that form quality tool set is the control chart. Control charts are efficient instruments for checking changes or variations in the processes. Throughout this tool is intended to get a model, which detects better the variations in the average, when it is necessary to control more sensitive processes.

The choice of the control charts to be used depends on the characteristics to be measured in the process, as well as the way that these samples are taken. There are situations in which the samples should be formed by an individual unit, because of its cost or operation. If the process is adjusted to control

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small variations and the sample consists of an individual unit it is recommended to use a control chart of Cumulative Sum or Exponentially Weighted Moving-Average control chart. As both are equally suggested, the purpose is to verify if they present similar results in all standard deviation range.

Many times, little attention is given to all dimensions of a process: cost, efficiency, productivity and quality. The improvement of effective quality can be an instrument for increasing productivity and reduction of costs. The installation of statistical process control and the consequent reduction of the variability results in a decrease of the manufacturing costs and an increase in productivity. This means an increase in production capacity, without any additional investment in equipment, workforce or overhead. In the control process, the data routinely collected are used and this information is employed in a practical way for the staff, engineers and managers to work on the process improvement. In this way the cost of implementing these improvements in quality and productivity is almost insignificant. One of the procedures applied, in this kind of confirmation is the control chart (Vaughn, 1990).

According to Dr Walter A. Shewhart, the control charts would be useful first, to define limits or the process prototype that the manager works hard to reach, second, they would be used as an instrument to get to the target, and third they would be used as a way to evaluate when the desired target is reached. Therefore, they are instruments used in the specification, production and inspection, and when they are used, they bring these three industry phases into a complete interdependence (Duncan, 1974).

The disadvantage of Shewhart control charts, is that they use only the information enclosed about the process in the last plotted point and they ignore information given by the sequence of all points. This feature makes Shewhart control charts, relatively insensitive to small changes in the process, in the order of 1.5 of standard deviation, or less. Naturally, other criteria can be applied to Shewhart schemes, such as sensitizing rules and the use of warning limits which attempt to incorporate information from all point sets in the procedure of decision making. However, the use of these supplemental sensitizing rules reduces the simplicity and the easiness of Shewhart control charts interpretation. Besides, the use of these sensitizing rules can reduce the average run length of the control chart, when the process is in control, before a false alarm signal, which would be undesired (Montgomery, 1996).

Shewhart control charts are efficiently complemented by CuSum (Cumulative Sum) and EWMA (Exponentially Weighted Moving-Average) control charts when there is interest in detecting small changes in the process. These methods are considered highly efficient in detecting special causes of variation, which lead to the non-conformity of production.

The literature concerning to the control of small changes in the process average the EWMA and CuSum control charts are recommended so, it is the purpose here to investigate which of them better detects an out of control signal in some range of standard deviation. The changes in the average, measured in magnitudes of standard deviations, can be proved through the alterations introduced in stable processes. The greatest contribution of this research is to determine which model should be used according to the variation magnitudes that must be detected.

Some authors, such as Duncan (1974), Hawkins and Olwell (1998) and Lucas (1976), state that the Cumulative Sum control chart is much more efficient than the usual Shewhart control charts, concerning to small variations in the average. Other authors, such as Crowder (1987 a, b) and Lucas and Saccucci (1990), presented the Exponentially Weighted Moving-Average control chart as a good choice to detect changes in little extension in the process average.

However, when such methods are exposed in the literature there is not a definition for a practical question: which of the two charts would have the best performance? That is, alternatively to Shewhart control charts there would be any criterion to choose one of the two charts presented? Which of them performs the task of better showing an out of control situation, CuSum or EWMA?

In order to answer these questions, it is proposed in this present study that, the Cumulative Sum (CuSum) and the Exponentially Weighted Moving-Average (EWMA) control charts are used, with applications to several series in control and several series with modification by small changes in the process average. The series were created following a Normal Distribution. The alterations in the average were determined in magnitudes of standard deviation. After the changes, the performance of each of the charts were compared, in order to verify if there are differences between them in detecting the changes introduced in the process.

The adopted criterion of measure to confirm the performance of a control chart is the ARL (Average Run Length). ARL has the function of determining how many samples are necessary so that the control chart presents the indicative signal of the out of control state. ARL_0 is the expected number of samples until a false alarm is given, when the process is in control.

2. Univaried control chart

The statistical process control (SPC) is formed by a set of tools to solve problems, in order to get the stability and improvement in the capacity of the processes through the reduction of the variability. These tools are technically important for SPC and include their technical aspects. Among these tools, control charts are the most sophisticated ones. (Vaughn, 1990).

The main purpose of the statistical process control is to quickly detect the occurrence of special causes of change in its execution way. Control charts can be used to estimate the parameters of a production process and, through this information, determine the process capacity. They can also give useful information for the improvement of the function sets related to production. The goal of statistical process control is to reduce the variability and the control charts are efficient tools to reduce this variability as much as possible (Vaughn, 1990).

Montgomery (1996) highlighted five reasons for the control chart popularity:

- Control charts are proven technique for the improving productivity. A program that uses control charts can reduce waste and the rework, which harm productivity in any operation. In this way there is a production increase and a cost decrease.
- Control charts are efficient in preventing faults, helping to keep the process in control. It is the philosophy of doing it right from the first time. It is more expensive to classify faulty and perfect items than manufacturing just good items. If there is not an efficient process, somebody is being paid to produce items of no good quality.
- Control charts prevent unnecessary process adjustments. A control chart can distinguish between a common cause and a special cause of variation. Unnecessary adjustments can result in a deterioration of the process development.
- Control charts provide diagnosis information. The point drawing shape that the control chart gets, will often contain information with diagnosis value for an experienced operator or engineer.
- Control charts produce information about the process capacity, through the value of their parameters and stability about the time. This allows an estimate of the process capacity. This information is of an extraordinary use for product and process engineers.

Specifying the control limits is one of the critical decisions that must be taken in the project of the control chart. Separating the control limits of the center line, reduces the risk of a Type I error. However, increasing the control limits, also increases the risk of a Type II error. If the limits for the center line are drawn closely an opposite effect is obtained: the risk of Type I error increases while the risk Type II error decreases. (Montgomery, 1996)

A generalized quality control procedure was proposed by Champ et al. (1991), where a control chart set was presented from a generalized chart and it was stated that, at this procedure, there are special cases of control charts: Shewhart $\bar{X}$-chart, the Cumulative Sum chart and the Exponentially Weighted Moving-Average chart. It was also proposed a chart that is a combination of CuSum and EWMA charts. In their point of view, this one seemed favorably comparable to both CuSum and EWMA charts.

2.1. Cumulative sum control chart (CuSum)

The cumulative sum control chart was initially proposed in England by Page (1954) and has been studied by many authors. Ewan (1963) outlined many schemes of control chart and the type of process which the CuSum charts are most appropriate. Bissel (1969) considered the CuSum method and its relevance to quality control. He proposed extensions of this technique to facilitate its application to practical situations. Goel and Wu (1973) presented a procedure for the economic project of CuSum chart to control the process average with a normally distributed quality characteristic. The technique is employed to determine the optimum values of the samples size, the sampling interval and the decision limit. Lucas (1973, 1976) proposed a modification of the V-mask control scheme for CuSum chart. Reynolds (1975) presented an approximation of the average run length for CuSum chart to signalize the points out of control. He used an analogy between the procedure of CuSum control chart with independent and identically distributed normal random variables and the procedure for CuSum control chart which did not require the normality assumption. Johnson and Bagshaw (1974, 1975) concluded that CuSum test was not powerful when the observations were non-independents. The major emphasis was given to the obtained effects on the average run length distribution caused by the presence of correlation. Hawkins (1981) presented a technique for employing the same CuSum procedure used on mean control for controlling the variance. Lucas and Crosier (1982) presented the average run length for CuSum chart signalized out of control points. They showed run length distribution in control schemes and out of control. They also presented ARL tables and figures showing the run length for CuSum control schemes with the Fast Initial Response (FIR) feature. Woodall (1985) presented a method for projecting quality control charts on the basis of their statistical performance over specified in control and out of control regions of parameter values. Vance (1986) developed a computer program in order to calculate the average run lengths of CuSum chart for controlling normal means.

In the present work, formulas and equations presented by Montgomery (1996) were used. He introduced the cumulative sum control chart, applied for monitoring process average and variability. He mentioned that it was possible to project CuSum procedures for other statistical variables, such as Binomial and Poisson variables for modeling non-conformities and continuous processes.

CuSum chart directly incorporates all the information in the sequence of sample values by plotting the cumulative sums of the sample values deviations from a value objective. Assuming that samples of size $n \geq 1$ are collected, $\bar{x}_j$ is the average of the jth sample and μ_0 is the value wanted for the process average, the CuSum control chart is formed by demarcating the formula (1) resulting quantity along with sample i

$$C_i = \sum_{j=1}^{i} (\bar{x}_j - \mu_0), \tag{1}$$

where C_i is the cumulative sum including the ith sample, since they combine information from several samples. Cumulative sum charts are more efficient than Shewhart charts in detecting small process changes. Besides, they are particularly more efficient with samples of size $n = 1$.

If the process keeps in control at the target value μ_0, the cumulative sums defined in (1) describe a random way with zero average. On the other hand, if the average changes to any value above $\mu_1 > \mu_0$, then an ascendant tendency will develop at the cumulative sum C_i. Reciprocally, if the average changes to some value below $\mu_1 < \mu_0$, the cumulative sum C_i, will have a negative direction. Considering this, if at the demarcated points a tendency up or down appears, it must be considered an evidence that process average change, and a search for the assignable causes must be done.

There are two ways of representing the cumulative sum charts, the algorithmic CuSum chart and the V-mask form of the CuSum.

Lucas (1973, 1976) presented a modified scheme for the V-mask. In this modified scheme he includes parabolic section to the V-mask to improve its performance in detecting large shifts of the mean from goal conditions. He showed ARL curves and concluded that the modified V-mask worked better than the V-mask where its ARL is longer for small deviations from the goal and shorter for large deviations from the goal.

The V-mask procedure. A procedure, which became popular after Barnard (1959), for the use of an algorithmic CuSum was the V-mask control scheme. Essentially, the same scheme was previously suggested by Page (1954). The V-mask is applied to successive CuSum statistic values

$$C_i = \sum_{j=1}^{i} y_j = y_i + C_{i-1}, \tag{2}$$

where y_i is the standardized observation $y_i = (x_i - \mu_0)/\sigma$.

Montgomery (1996) used Johnson's method and adapted the actual values of ARL_0 for a V-mask projected scheme. At this scheme the in control ARL should be $\mathrm{ARL}_0 = 1/2\alpha$. In order to result in ARL_0 of 370, α value must be equal to 0.00135 and in ARL_0 of 500 the value of α is 0.001.

2.2. The exponentially weighted moving-average control chart, EWMA

Montgomery (1996) presented the exponentially Weighted Moving-Average, or EWMA, as a good choice when the interest is in detecting small changes in the process. The performance of the EWMA control chart is approximately equivalent to that of the cumulative sum control chart.

EWMA control chart was introduced by Roberts (1959), but there are many authors who presented good contributions to this kind of control chart. Hunter (1986) said that the differences among Shewhart, CuSum and EWMA control charts have to do with the way each charting technique uses the data generated by the production process. He illustrated in its simplest form how the charts weigh the data. Shewhart chart depends entirely on the last demarcated point. CuSum chart attributes equal weight to the most ancient datum as well as the most recent. EWMA gives higher weight for more updated information and lower weight for more remote information. Woodall and Maragah (1990) cleared that the usual V-mask control rule has the effect of ignoring some of the past history of the process. Therefore, the uniform weight is given only to a random number of observations which are more recent.

Crowder (1989) distinguished between two different uses of the EWMA, that of forecasting future observations from process with drift, and that of monitoring process subject to occasional shifts in mean level. For the second use he motivated the use of EWMA graphically, illustrating how the EWMA provides a clearer idea of process shifts, and produces smaller ARLs than the traditional $\bar{X}$ chart. Ng and Case (1989) presented methodologies to construct control charts using the EWMA of a sample statistic for subgroups of size n and individual date. It was also presented consistent and systematic approach for deriving control charts of the EWMA to monitor the mean and dispersion process.

Lucas and Saccucci (1990) described the properties of EWMA control schemes and compared them with CuSum control schemes. The results showed that the properties of EWMAs are very close to those of CuSum. According to the presented table, EWMA chart shows ARL slightly smaller than the algorithmic CuSum chart. However, at the complementary discussions Woodall and Maragah (1990) told that EWMA can be slower to react than CuSum chart, for some changes in the process.

Montgomery (1996), was also one of the authors who studied this subject and defined the Exponentially Weighted Moving-Average as

$$Z_i = \lambda x_i + (1 - \lambda)Z_{i-1}, \tag{3}$$

where $0 < \lambda \leq 1$ is a constant and the starting value (required with the first sample at $i = 1$) is the process target, so that $Z_0 = \mu_0$. Sometimes the average of preliminary data is used as the starting value of the EWMA, so that $Z_0 = \bar{x}$.

The center line and the control limits for the EWMA control chart are:

$$\text{LCS} = \mu_0 + L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}[1-(1-\lambda)^{2i}]} \tag{4}$$

$$\text{LC} = \mu_0 \tag{5}$$

$$\text{LCI} = \mu_0 - L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}[1-(1-\lambda)^{2i}]}. \tag{6}$$

In Eqs. (4) and (6), the factor L is the width of the control limits. The term $[1-(1-\lambda)^{2i}]$ approaches unity as i gets larger. Montgomery (1996) elucidated this means that after the EWMA control chart has been running for several time periods, the control limits will approach steady-state values given by:

$$\text{LCS} = \mu_0 + L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}} \tag{7}$$

and

$$\mathrm{LCI} = \mu_0 - L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}} \tag{8}$$

Montgomery (1996) recommends to use the control limits (7) and (8) for small values of i. This greatly improves the performance of the control chart, in detecting an off-target process immediately after the EWMA is started up.

Design of an EWMA control chart. The EWMA control chart is very efficient in the situations where small changes happen in the process. The chart design parameters are the multiple of sigma used in the control limits (L) and the value of λ. To detect small changes, it is possible to choose these parameters to give ARL performance the EWMA control chart, that closely approximates CuSum ARL performance.

Several studies were developed about average run length of the EWMA control chart. Robinson and Ho (1978) used a numerical procedure to determine ARL, presenting tables for various settings of the control limits smoothing constant and shift in the nominal level of the process mean. Crowder (1987b) presented a numerical procedure using integral equations for the moments tabulation of EWMA charts run lengths. Both average run lengths and standard deviations of run lengths were presented for the two-sided EWMA chart assuming normal observations. Crowder (1987a) presented a computer program that calculates the ARL of EWMA chart for controlling the normal process average. Lucas and Saccucci (1990) evaluated the properties of an EWMA control scheme used to monitor the average of a normally distributed process that may experience shifts away from the target values not commonly used in the literature shown to be useful for detecting small shifts in a process.

In practice, values of λ which work well in the interval $0.05 \le \lambda \le 0.25$, are found, being $\lambda = 0.05$; $\lambda = 0.10$ and 0.20 popular choices. The usual three-sigma limits ($L = 3$) work reasonably well particularly with the larger value of λ. Even when λ is small, ($\lambda \le 0.1$), there is an advantage in reducing the width of the limits, using a value of L between 2.6 and 2.8 (Montgomery, 1996).

Hunter (1986) suggested choosing λ so that the weight given to current and previous observations matches as closely as possible the weights given to these observations by a Shewhart chart. This results in a recommended value of $\lambda = 0.4$. A procedure suggested by Montgomery (1996) to the optimal design procedure would consist of specifying the desired in-control and out-of-control average run lengths and the magnitude of λ and L that provide the desired ARL performance. He also showed tables with results for different values of λ and L, which indicate that the chart would have $\mathrm{ARL}_0 \cong 500$. These values are, respectively: $\lambda = 0.4$ and $L = 3.054$; $\lambda = 0.25$ and $L = 2.998$; $\lambda = 0.2$ and $L = 2.962$; $\lambda = 0.1$ and $L = 2.814$; $\lambda = 0.05$ and $L = 2.615$.

3. Description of the methodology used

To reach the main purpose of this research the data were generated from the standard deviation average for the industrial oven temperature of special ceramic. The data used to develop this research were generated using a real data set composed of data collected at every one hour, in three different process phases: warming, burning and cooling zone subdivided in four moments of sampling, during 15 days, covering a total of 1.440 data. From these data the global average and the process standard

Table 1
Standard deviation fractions to determine the changes in the process global change and the respective altered averages (process global average: 908.1598; standard deviation: 22.9212)

Changes (+)		Changes (−)	
Standard deviation fractions (change magnitude)	Altered average	Standard deviation fractions (change magnitude)	Altered average
1.5	942.542	−1.5	873.778
1.375	939.676	−1.375	876.643
1.25	936.811	−1.25	879.508
1.125	933.946	−1.125	882.373
1.0	931.081	−1.0	885.239
0.875	928.216	−0.875	888.104
0.75	925.351	−0.75	890.969
0.625	922.486	−0.625	893.834
0.5	919.620	−0.5	896.699

deviation was taken. Using the average and the standard deviation a new data set were generated, following a normal distribution with 100 values each.

With the global average and the process standard deviation, the changes in magnitudes of standard deviation were also calculated. From the wanted variations, using standard deviation fractions, new average values were determined with the purpose of introducing the instability in the system. Applying these standard deviation fractions on the global average, the altered averages, whose values are found at Table 1, were obtained.

Using the altered averages, according to Table 1, and the process standard deviation new series were randomly generated with the purpose of introducing them in the series of one hundred samples. Some care was taken regarding the samples to form these new series. Each set of 20 values was selected in a way that the samples exhibited, at the most, the intended variations corresponding the change to be detected in each item. It was also given attention so that the first sample already contained the desired difference, making sure, this way that such sample served as basis for all the indicated alterations.

To guarantee that all series were in control an x-bar chart was used. After, it was evidenced that the series did not signalize any points out of control, they were said to be in control. The new series of 20 samples generated with the values of Table 1, were introduced on the in-control series by the following procedures. For each standard deviation fractions (change magnitude) proposed was used only one series, which suffered alterations in 10 different positions. The first alteration was introduced from the 1st to the 20th position. The second alteration began at the 11th sample and it went up to the 30th one. The third alteration was from the 31st to the 50th in successively, in a way that the last alteration was designated for the samples 91–100.

Making use of the series in control preceded the alterations according to the positions strategically defined. The altered series was evaluated again by Shewhart control chart for individuals. It was established that, by the sensitizing rules, the changes introduced signalized the samples out of control in every accomplished alteration. See illustrations with change magnitude of 1.0 standard deviation according to Fig. 1.

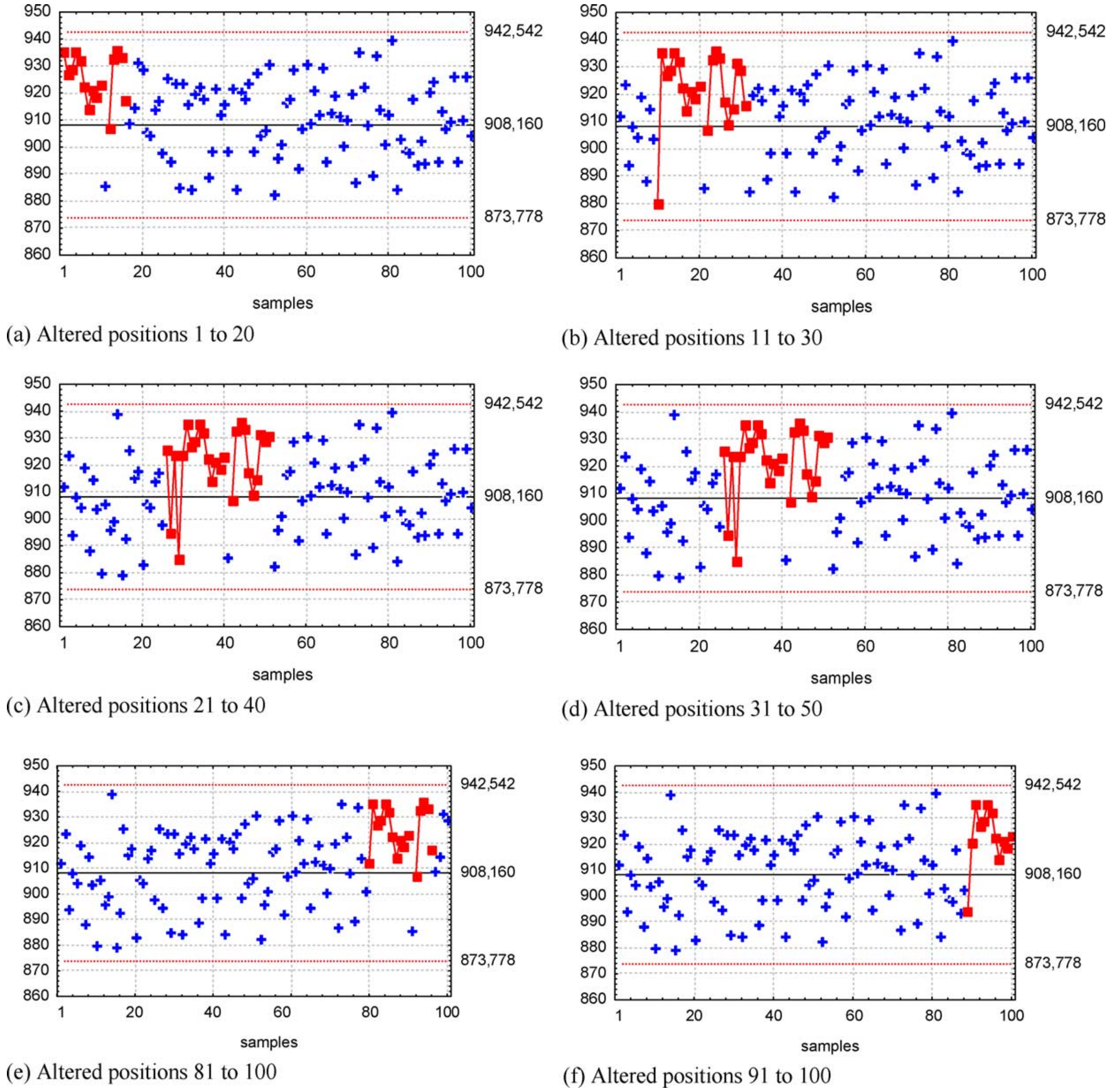
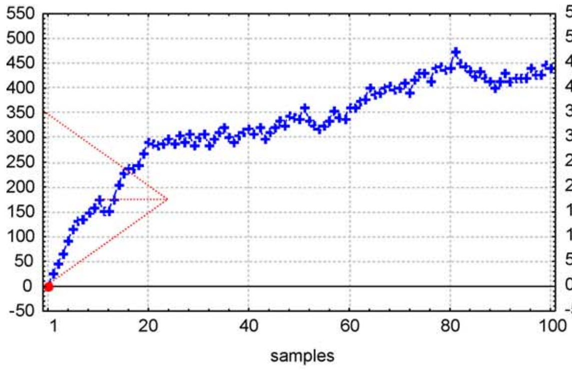


Fig. 1. Series out of control altered with change magnitude +1 standard deviation.

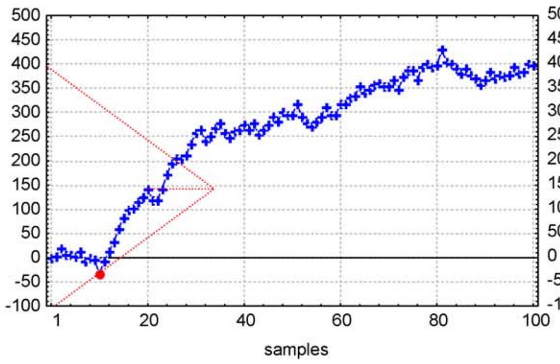
3.1. Application of CuSum and EWMA control charts

After the alterations were introduced sequentially, the series were analyzed using CuSum and EWMA control charts. EWMA control chart with different values for the parameters specifications λ (smoothing constant), and L (control limit width according to the standard deviation).

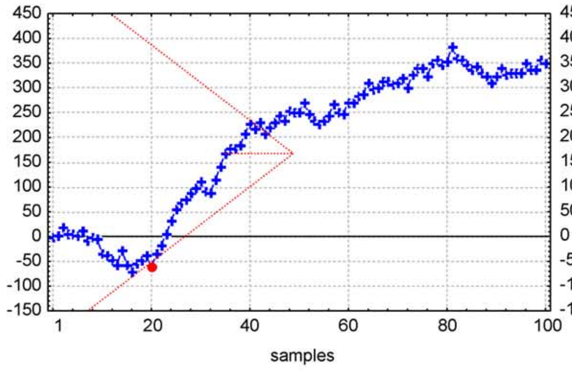
In order to illustrate the procedures, the resultant charts of the change magnitudes of $+1$ and -1 standard deviation is shown in sequence for both CuSum and EWMA control charts according to Figs. 2 and 3.



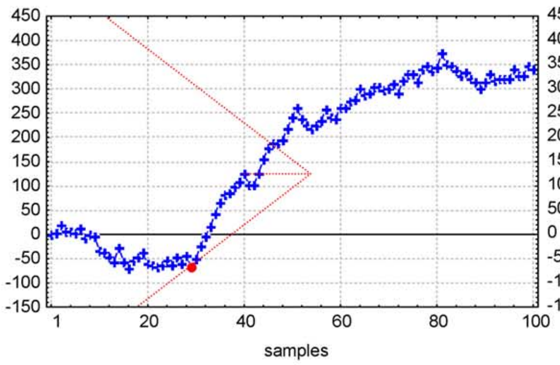
(a) Altered positions 1 to 20



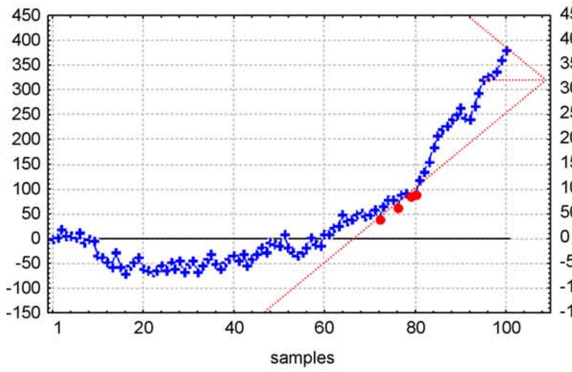
(b) Altered positions 11 to 30



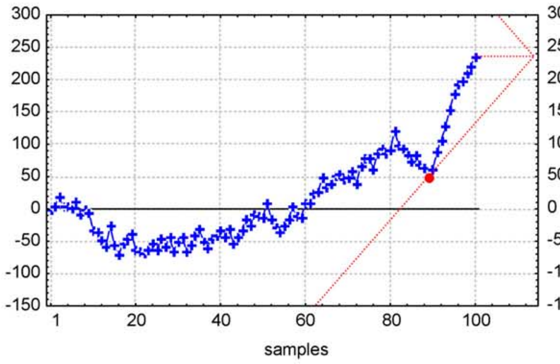
(c) Altered positions 21 to 40



(d) Altered positions 31 to 50



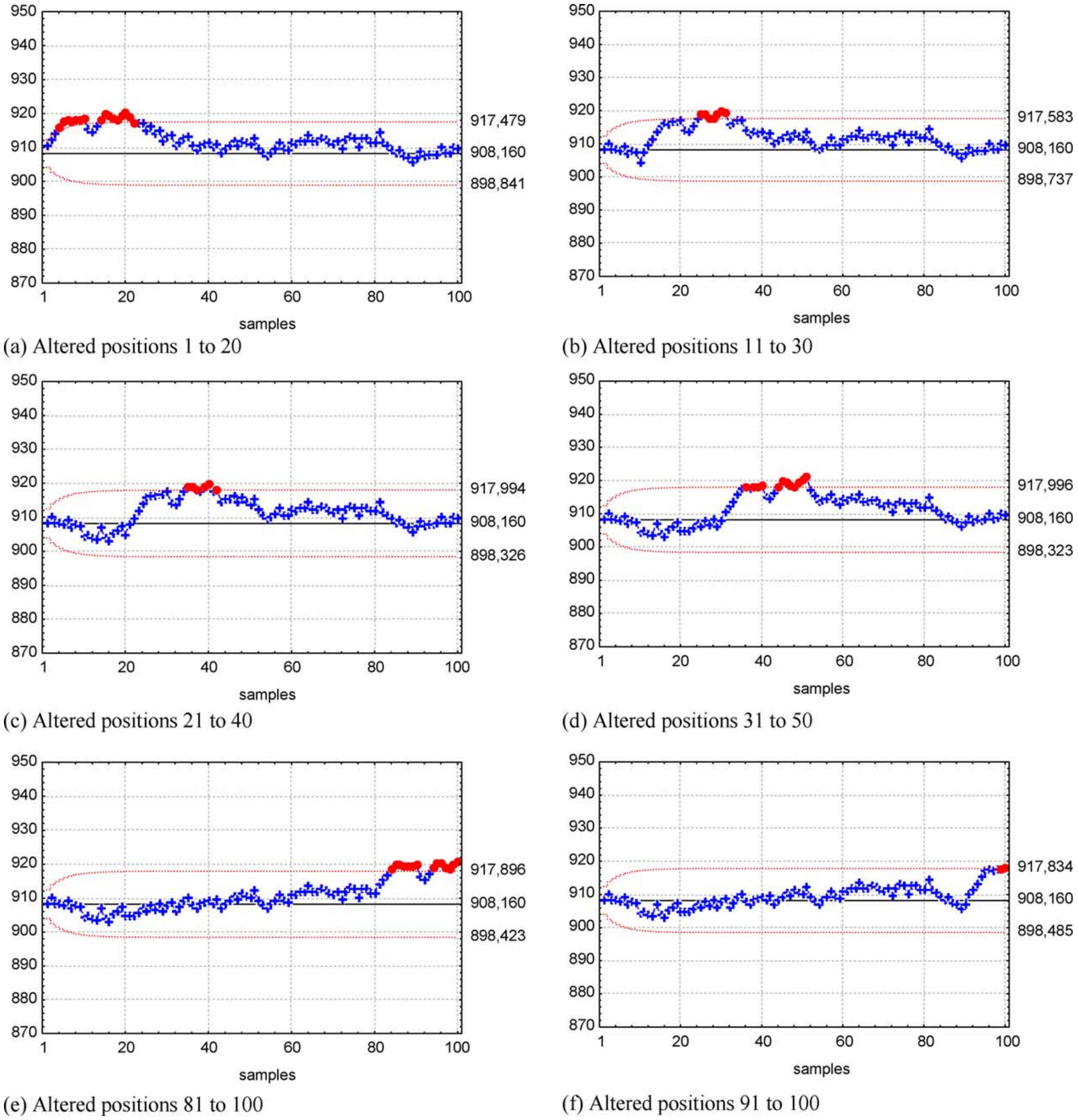
(e) Altered positions 81 to 100



(f) Altered positions 91 to 100

Fig. 2. Cusum control chart to the altered series with the change magnitude of $+1$ standard deviation.

The specifications used in each chart followed what was exposed in this paper, adopting as a comparative measure, the expected number of samples taken before an out of control signal was given for a stable process, the ARL_0. For CuSum control chart the support was given by Johnson's method. Following recommendations given by Montgomery (1996) for ARL_0 of EWMA chart, the values of constant λ and the respective control limit width L were chosen. For the accomplished analysis, CuSum



(a) Altered positions 1 to 20
(b) Altered positions 11 to 30
(c) Altered positions 21 to 40
(d) Altered positions 31 to 50
(e) Altered positions 81 to 100
(f) Altered positions 91 to 100

Fig. 3. EWMA control chart with the parameters $\lambda = 0,10$ E $L = 2814$, to the altered series with the change magnitude of $+1$ standard deviation.

and EWMA control chart were applied with the specifications that would result in an ARL_0 of 500 for both charts.

The Statistic program, following the below specifications for each of the chart, extracted the results presented in tables:

- CuSum, V-mask, with the superior and inferior control limits defined by the change to be detected in standard deviation fractions, adjusted for each analysis with the average of 908.1598 and data computed standard deviation. Probability of Type I error, $\alpha = 0.001$ and probability of Type II error, $\beta = 0.001$.
- EWMA with the average 908.1598 and the data calculated standard deviation. The values for the constant λ, and the respective control limit width considering standard deviation L.

4. Result analysis

The changes in the average, proposed according to Table 1, constituted several items of study. For each variation aimed CuSum and EWMA control charts were executed, similar to the changes with magnitude of $+1$ and -1 standard deviation shown in Figs. 2 and 3. Each result set, obtained from the respective control charts, was exposed in tables. Their contents refer to the average run length from the alteration until appearing the first out of control point, the ARL given by CuSum and EWMA control charts specified for the parameters λ and L proposed for this study.

In each column, referring to CuSum control chart and/or the respective variation in the specification of EWMA control chart, the average in ARL was calculated. In the average determination, it was considered the positions that exhibited points out of control. These averages formed the essential part for the comparisons and analysis of each chart performance. The smallest average resulting in ARL was the criteria used to elect the most efficient chart or model.

Following the presented steps in the preceding item, the analysis for all the proposed variations were accomplished. To give examples, the results found with the changes with magnitudes of $+1$ and -1 standard deviation shown up in Tables 2 and 3.

Table 2
Average run length for CuSum and EWMA charts with the variation of $+1$ standard deviation

Alteration	CuSum	EWMA				
		$L = 3.054$, $\lambda = 0.40$	$L = 2.998$, $\lambda = 0.25$	$L = 2.962$, $\lambda = 0.20$	$L = 2.814$, $\lambda = 0.10$	$L = 2.615$, $\lambda = 0.05$
01–20	−1	4	3	3	3	2
11–30	−1	–	–	14	14	9
21–40	−1	–	–	–	14	14
31–50	−2	–	4	4	5	13
41–60	−4	–	3	3	4	7
51–70	−4	–	4	4	4	4
61–80	−2	–	3	3	3	4
71–90	−1	–	3	3	3	3
81–100	−9	–	4	3	3	3
91–100	−2	–	4	4	8	7
Average	−2.7	–	3.5	4.56	6.1	6.6

Table 3
Average run length for Cusum and EWMA charts with the variation of − 1 standard deviation

Alteration	CuSum	EWMA				
		$L = 3.054$, $\lambda = 0.40$	$L = 2.998$, $\lambda = 0.25$	$L = 2.962$, $\lambda = 0.20$	$L = 2.814$, $\lambda = 0.10$	$L = 2.615$, $\lambda = 0.05$
01–20	–	–	7	7	5	4
11–30	–	–	5	5	5	5
21–40	–	–	5	5	5	5
31–50	–	–	7	7	7	7
41–60	–	–	7	7	7	7
51–70	–	–	–	7	7	7
61–80	–	–	7	5	5	5
71–90	–	–	7	5	7	7
81–100	−2	–	5	5	5	6
91–100	−3	–	5	5	5	7
Average	−2.5	–	6.1	5.8	5.8	6.0

4.1. Changes with the variation of +1 standard deviation

In order to expose the results of all the altered positions, for CuSum chart as much as for EWMA chart with the respective values given λ and L, Table 2.

The development of CuSum control chart was better than any of EWMA chart specifications. The alteration encompassing the samples 81–100 showed the signalization a lot before the introduced changes. Investigating the data series close to this point, a variation of 1.18 standard deviation was verified in sample number 73. In this case, the signal was given one position before.

EWMA control chart, with constant $\lambda = 0.40$, did not signalize in 90% of the charts, representing the least indicated choice. EWMA control chart, specified by $\lambda = 0.25$ and 0.20, presented its performance in a similar way, except for the alteration 11–30, signalized by $\lambda = 0.20$ and not signalized by $\lambda = 0.25$. These parameters, although resulting in an inferior averages, when compared to the other two did not present good work, once some graphs did not alert the out of control situation. The other two specifications of EWMA chart ($\lambda = 0.10$ and 0.05) presented the signal with bigger oscillation from one graph to the other, but detected all the alterations. EWMA control chart specified by the parameters $\lambda = 0.10$ and $L = 2.814$ presented the best result.

For the average calculation, it was taken into account the difference found between the position of the first signalized sample and the introduced alteration, the ARL, as it was shown in Table 2. Comparing each result found by EWMA chart ($\lambda = 0.10$ and $L = 2.814$) and the respective graphs of CuSum chart, it was observed that, in average, CuSum chart signalized faster than EWMA chart 8.8 samples.

4.2. Change with the variation of − 1 standard deviation

The same way CuSum and EWMA control charts were employed to the altered series in −1 standard deviation. In order to synthesize the information given by the control graphs Table 3 was built.

Table 3 shows that CuSum control chart did not signalize, except for the two latter altered positions. The chart EWMA, of constant $\lambda = 0.40$, did not detect the change in any of the positions. EWMA chart that presented smaller ARL, was the one with constants $\lambda = 0.20$ and 0.10. With these parameters EWMA chart resulted in the same final average. They distinguished only in the alterations located in the samples 01–20 and 71–90. EWMA chart specified by $\lambda = 0.10$ and $L = 2.814$ detect initial alterations more quickly and because of that it was considered better for the analysis of this level of variation.

4.3. Final results

The final results of all altered positions for the several change magnitudes (-1.5σ to 1.5σ) were determined in a similar way as shown in Tables 2 and 3. With the results obtained the ARL average of each control chart was organized the Table 4. The first column of this table represents the variations introduced in the average, in standard deviation fractions. The other columns are formed by ARL and the number of positions signalized by each of the given charts by the respective CuSum and EWMA control graphs. The bold-faced values represent the chart that presented the best performance.

Analyzing Table 4, the line between +1 and −1 standard deviation was observed to be different. The positive changes (0.5–1.0) signalized more times than the negative changes (−0.5 to −1.0). The results

Table 4
The result summary when the first sign out-of-control appears for CUSUM and EWMA control charts for the standard deviation fractions from -1.5σ to 1.5σ

Change Magnitude	CUSUM		EWMA $L = 3.054$, $\lambda = 0.40$		EWMA $L = 2.998$, $\lambda = 0.25$		EWMA $L = 2.962$, $\lambda = 0.20$		EWMA $L = 2.814$, $\lambda = 0.10$		EWMA $L = 2.615$, $\lambda = 0.05$	
	*	**	*	**	*	**	*	**	*	**	*	**
1.5	−**1.0**	**10**	6.3	10	3.9	10	**3.8**	**10**	4.4	10	5.2	10
1.375	−**1.5**	**10**	3.8	10	**3.7**	**10**	3.9	10	6.4	10	7.11	10
1.25	−**2.3**	**10**	3.8	10	**3.2**	**10**	**3.2**	**10**	3.8	10	4.6	10
1.125	**1.89**	**9**	–	–	12.78	9	**10.89**	**9**	11.1	9	12.0	9
1.0	−**2.7**	**10**	4.0	1	3.5	8	4.56	9	**6.1**	**10**	6.6	10
0.875	−13	**1**	–	–	4.0	1	3.33	3	4.75	4	**6.57**	**7**
0.75	−**3.22**	**9**	–	–	13.0	3	11.78	**9**	**8.33**	**9**	8.89	9
0.625	−6	1	–	–	–	–	–	–	19.6	8	**16.3**	**10**
0.5	–	–	–	–	–	–	–	–	10.5	2	**10.67**	**3**
−0.5	–	–	–	–	–	–	–	–	–	–	**3.0**	**1**
−0.625	–	–	–	–	–	–	–	–	–	–	–	–
−0.75	–	–	–	–	–	–	–	–	–	–	–	–
−0.875	–	–	–	–	–	–	–	–	–	–	–	–
−1.0	−2.5	2	–	–	6.1	9	5.8	10	**5.8**	**10**	6.0	10
−1.125	−**2.3**	**10**	7.0	10	4.5	10	**4.3**	**10**	4.6	10	5.5	10
−1.25	−**2.0**	**10**	12.1	10	5.6	10	**5.3**	**10**	**5.3**	**10**	5.9	10
−1.375	−**1.7**	**10**	5.5	10	**3.6**	**10**	4.1	10	4.8	10	5.1	10
−1.5	−**2.7**	**10**	4.9	10	3.4	10	**3.2**	**10**	3.5	10	4.0	10

(*) Necessary number of the altered position samples to the position of the first sign; (**) number of signalized positions.

obtained by the CuSum control chart was shown slightly different when compared to small positive changes with the small negative changes for the same average extent levels. At these levels CuSum control chart, almost did not signalize, showing themselves less efficient than EWMA control chart, except for two items (1 and 0.75), whose positive variations were detected with better efficacy by CuSum chart.

For these changes (less than 1 standard deviation) the number of positions signalized by CuSum control chart and by their respective parameters specified for EWMA chart was compared using the hypothesis tests using difference between two proportions. The CuSum control chart signalized for 23 positions and EWMA chart signalized, respectively, for 50 and 43 positions. It was concluded at a significance level of 5% that EWMA control chart to the parameters $\lambda = 0.050$; $L = 2.615$ and $\lambda = 0.10$ and $L = 2.814$ differ significantly from the CuSum control chart presenting better performance for the average change with magnitudes smaller than 1 standard deviation.

Making a parallel for the results found by CuSum chart and by EWMA chart, defined with the parameters that were more efficient for ARL, the average changes were determined, according to what was exposed in Table 5.

Applying the tests of hypotheses to means of normal distributions, variance unknown, at a significance level $\alpha = 0.05$, it can be concluded that the ARL of CuSum chart is significantly smaller than EWMA chart.

Table 5
Average differences of expected samples number up to when the first out of control signal appeared between Cusum and EWMA control charts

Change magnitude	ARL for the best control chart		Differences
	CuSum	EWMA	
1.5	−1.0	3.8	4.8
1.375	−1.5	3.7	5.2
1.25	−2.3	3.2	5.5
1.125	1.89	10.89	9.0
1.0	−2.7	6.1	8.8
0.875	–	6.57	–
0.75	−3.22	8.33	11.55
0.625	–	16.3	–
0.5	–	10.67	–
−0.5	–	3.0	–
−0.625	–	–	–
−0.75	–	–	–
−0.875	–	–	–
−1.0	–	6.0	–
−1.125	−2.3	4.3	6.6
−1.25	−2.0	5.3	7.3
−1.375	−1.7	3.6	5.3
−1.5	−2.7	3.2	5.9
Average	−1.7	6.3	7.0

5. Conclusion

Montgomery (1996) argued that, in order to accomplish the total quality control program, key elements are necessary, such as the quality philosophy, the costs, the legal aspects and the execution. Although the statistics practices are technical tools for the quality control and improvement, to be used with more efficiency, they must be part of the management system that is leading to quality and have the guaranty of its execution in all business aspects. The concept of Total Quality Administration is an administrative structure that implements statistical methods.

Charts or graphs are one of the main techniques of the statistical process control. Their use is favorable for the quality monitoring performance. When non-common variation sources are presented, points out of the control limits or some sequence form or some tendency can appear. This is a sign that investigations should be done in the process taking the decision making for the corrective action in the sense of removing the variability sources. The systematical use of the control chart is an excellent way of reducing the variability.

Control charts produce information about the process capacity through their parameters and their stability on the time. This allows to estimate the process capacity. This information is of specific use of process and product engineers.

Control charts, classified as Shewhart charts, are among the most important and useful techniques in the statistical process control. The basic rule in the use of these charts is to take action when a point is beyond the usual three-sigma limits. Other criteria were developed, such as the use of warning limits and the sensitizing rules, with the purpose of improving their performance in detecting small process changes. From these developments, a natural step was to adopt a rule to take action, which would be based, in all data and, not only in the last samples.

Two complementary options show up to be used when the interest is to detect small variations, the Cumulative Sum control chart (CuSum) and the Exponential Weighted Moving-Average (EWMA). Both are equally recommended for the quality control performance in these situations.

CuSum chart type, used in the analyses is the procedure of V-mask, for being the most popular and available in computer programs. The specifications adopted for the respective CuSum and EWMA control charts, originate from the orientations given to get to the expected number of samples in the process in control, the ARL_0 of 500, for both charts.

Making use of the Cumulative Sum chart and Exponentially Weighted Moving-Average, several sets of samples were analyzed and are called series. These series were declared in control and altered purposely, in different moments with variations in the average determined by magnitudes of standard deviation.

After the application of CuSum and EWMA control charts, in a general way, it was possible to notice that CuSum control chart was more efficient in all analysis accomplished with the changes in the order of $+1$ standard deviation or more, and for all the alteration in the order of -1.125 standard deviation or less.

Among the established specifications for EWMA control chart, it was observed that two of them detect the biggest number of positions for smaller than 1 standard deviation change. These were the ones with constants of $\lambda = 0.10$ and 0.05 with the respective width of control limits $L = 2.814$ and 2.615. Therefore, these were thought to be the most suitable parameters for these situations.

After the analysis accomplishment a generalized answer was found to the problem. In order to detect the average changes, at the order of 1 standard deviation or less, the control chart that had a better performance was EWMA, specified by $\lambda = 0.10$ and 0.05 with the respective control limits $L = 2.814$ and 2.615. For bigger changes of 1 standard deviation but smaller than 1.5 the one that did a better job was CuSum.

EWMA chart with the parameters $\lambda = 0.40$ and $L = 3.054$, did not signalize in any of the change levels presented. Hunter (1986) recommended the value of $\lambda = 0.40$, as a suggestion, so that the weight given to the observations now and before were paralleled, as much as possible to the weight given to the observations of a Shewhart chart.

The purpose of verifying if, in the process, the variation happenings would be detected at the same speed, the location for the introduced changes were strategically chosen, being the positions: 1st, 11th, 21st and so on. This strategy was used because, in practice, the moment when the oscillation happens is not known. Analysis showed that CuSum control charts, in a general way, present the same performance and it does not matter if the change happened at the beginning, in the middle or at the end of the series.

EWMA control charts showed that the changes that happen at the beginning of the series are more rapidly detected. This is in accordance with what was recommended by Montgomery (1996), for the use of small values for the sample of position i (or time).

Montgomery (1996) explained that although EWMA is presented as a statistical process monitoring tool, it really has a much wider interpretation. From the viewpoint of the statistical process control, EWMA control chart is comparable to CuSum control chart in its capacity of monitoring a process and detecting the presence of assignable causes, which result in changes. However, EWMA produces the forecasting of where the average will be in the next period of time what makes it easy to apply in the industry. This makes EWMA a more powerful tool.

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