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Multivariate feed back control: an application in a productive process

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Abstract

The main purpose of this research is to implement a multivariate feedback adjustment proportional to the last deviation from the target, in the set of variables wandering around the target. To apply the controller equation it will be necessary to study the exponentially weighted moving average (*EWMA*) statistic, in order to determine the behavior of the target disturbances. To determine the forecast values of the variables we will use Seemingly Unrelated Regression (*SUR*), which is necessary since there is a relationship between the variables and between the errors. In this manner, multivariate feedback adjustment can be reached based on scientific grounds.

Keywords: Multivariate feedback control; Seemingly unrelated regression; Engineering process control; Statistical process control; Proportional feedback; Monitoring

1. Introduction

Feedback control systems are used in many industries and with various kinds of equipment. A feedback system is the process of measuring an input variable, which is used to change the value of an output variable, while output measurements are used to determine the manner in which one can manipulate the input variables—hence, the name feedback. Feedback controllers are usually of the PID (Proportional-Integral-Derivative) type because they are standard industrial components. Yet despite the advantages, the problem of tuning PID controllers is a field in which much research can still be carried out (Lee & Kim, 2000).

According to Astrom and Wittenmark (1989), the main idea of a self-tuning (ST) controller is to separate the estimation problem from the control problem, since in most controllers there is an

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estimation algorithm that provides the estimated parameters to be used in the controller. The controller uses these estimates made from the process, carrying out the necessary adjustments. If the estimation is done independent of the control action, it is possible to do it externally from the production line, and whenever the variables are known ahead of time, one can decide which restrictions should be applied to the control variables.

Statistical Process Control (SPC) uses the measurements obtained in the process to carry out its monitoring and find changes that might be occurring, without, however, prescribing a control action. On the other hand, Engineering Process Control (EPC) uses the measurements obtained from the process that reveal its behavior, which allows for the prescription of changes in variables involved in the process, in order to make them the closest possible to the desired target. The joint use of these two methodologies provides an efficient way of controlling the quality of products and services.

Researchers such as Box, Hunter, and Hunter (1978), Box and Kramer (1992), Box and Luceño (1997), Del Castillo (1996), Mac Gregor (1987), and Ramirez (1994) have presented several studies and ways of carrying out the feedback adjustment, using a set of historical data that allows one to determine the future behavior of the series. Most studies use the techniques cited above, applied to sets of univariate data.

In this study we use a set of multivariate techniques for controlling the processes that have several characteristics to be monitored and/or feedback, and which most of the time are treated as independent variables. To achieve this objective we used a controller proportional to the error, since a commonly found problem with this type of controller is that it needs to be adjusted or synchronized each time operating conditions, specifications or some external factors change. The controllers regulate the process, acting directly on the variables that will influence the final characteristics of the product, such as temperature, flow of current, pressure and time for remaining in each stage of the process.

The statement that '…the objective of any control system is to adjust variables to meet the defined objectives of the process in regards to disturbances, using measurements of the variables…' Ramirez (1994) is true when applied to a set of univariate data. This treatment can lead to incorrect decisions if it is applied to a set of variables, for the univariate estimation does not consider the interaction of the variables when estimating their equations. To implement a multivariate controller one must consider the autocorrelation within and between the noise terms, in this case multivariate moving average (MA) noise, and within and among the responses Del Castillo (1996).

The main purpose of this article is to demonstrate how to achieve multivariate feedback control using a controller proportional to the last error, in a production process with interrelated variables and correlated errors. The vector autoregressive (VAR) model will be used to understand the relationship between the variables, and to determine the gain that each variable will provide to the system.

After a detailed study of the techniques used in this study, we applied the proposed methodology in a ceramic materials company that produces several types of tiles. The data were collected in a tunnel oven, where it was possible to observe 12 temperature variables, each one composed of 92 observations. The latter are pre-established according to the norms used by the company for burning ceramic materials. Next, we identified the variables of major interest in the system, estimating the regression equations, which were then used in the proposed feedback process, maintaining in this manner the process close to the stipulated target value.

2. Multivariate proportional feedback adjustment

A state of statistical control implies a random variation around the target value, caused by a wide variety of common causes. Even though effort is made to maintain the process close to target, there are several variables that cannot be controlled, such as room temperature, differences in raw materials, differences between production lots, equipment or machinery wear-and-tear, and even differences between operators, such as humor and satisfaction. These variables influence the process and are very difficult to eliminate, and most of the time it is economically unfeasible to do so; in these circumstances, an adjustment system is necessary (Box, 1991).

EPC has been discussed by several authors, such as Box and Luceño (1997), Box (1994), Box et al. (1978), Box, Jenkins, Reinsel (1994), Del Castillo (1996), Montgomery, Keats, Runger and Messina (1994), and Montgomery and Mastrangelo (1991), and Sachs, Hu, and Ingolfsson (1995). EPC is based on estimation methods of the variables involved in the process, since determining future behavior makes it possible to carry out the necessary system adjusting and tuning, thus maintaining stability.

With system feedback, corrections inputted into the system can be quantified, in order to be applied. Such measures can be carried out obeying the expression (1), known as PI (proportional-integral) controller because the control action can be reached whether action is applied using the proportional or the integral part.

$$gX_t = k_0 + k_P e_t + k_I \sum_{i=1}^{t} e_i, \tag{1}$$

Where X_t represents the variable that will undergo adjustment, which will have an effect of g units in the system, designated system gain. The gain can be determined through physical properties of the variable or curve adjustments, such as regression. The constants k_P and k_I correspond to the proportion in which each term of the controller will contribute to the linear combination (Box, 1991; Box & Luceño, 1997). The deviations of the target of a variable are represented by $d_t = X_t - A$, where X_t represents the variable that is being analyzed and A is the target value for this variable.

In the feedback control scheme, in periods prior to time t—the time when the action will happen—there are forecasted disturbance errors, represented by $\ldots e_t, e_{t-1}, e_{t-2}\ldots$, that determine the level at which the input variable X_t should be manipulated in order to achieve the minimum errors possible. In fact, what should be done, whenever possible, is to cancel the disturbance d_t through adjustment of the variable X_t, so that the process stays close to target. Therefore, Eq. (2) is written

$$X_{t+1} - A = d_{t+1} + gX_t. \tag{2}$$

This relationship shows that in the instant t, the deviation of the target $X_{t+1} - A$ depends on the disturbance d_{t+1} and on the adjustment level gX_t that the variable X suffered in the instant t.

Relationship (2) in the instant t shows that if one wanted to adjust X_t so that the right hand side of Eq. (2) would become zero, there would be no deviations from the target in the instant $t+1$ and $\mathbf{Z}_{t+1} - A$ would be zero. Unfortunately, this cannot be accomplished, since in the instant t we do not know the value of d_{t+1}. However, in the instant t the disturbance $\hat{d}_{t+1}$ can be forecasted, and one can write that $e_{t+1} = d_{t+1} - \hat{d}_{t+1}$ is the forecast disturbance error. Therefore, relationship (2) can be written as in Eq. (3)

$$Y_{t+1} - A = e_{t+1} + \hat{d}_{t+1} + gX_t. \tag{3}$$

In fact, it is necessary to adjust X_t so that

$$gX_t = -\hat{d}_{t+1}, \tag{4}$$

Substituting Eq. (4) in Eq. (3), we have

$$Y_{t+1} - A = e_{t+1}. \tag{5}$$

This relationship shows that the deviation from the target in the process will be the forecasted disturbance error. Relationship (4) shows the adjustment done in the instant t, but if the adjustment has to be carried out in a prior instant, it is had that

$$g(X_t - X_{t-1}) = -(\hat{d}_{t+1} - \hat{d}_t), \tag{6}$$

Yet it is known that $(\hat{d}_{t+1} - \hat{d}_t)$ does not mean the value of the disturbance, but instead the error that was made in forecasting the disturbance. This difference can be modeled according to exponentially weighted moving average (EWMA) statistics, assuming the form shown in Eq. (7).

$$(\hat{d}_{t+1} - \hat{d}_t) = \lambda(d_t - \hat{d}_t) = \lambda e_t. \tag{7}$$

Substituting Eq. (7) in Eq. (6), the expression of adjustment is obtained, represented as

$$g(X_t - X_{t-1}) = -\lambda e_t, \tag{8}$$

where e_t represents the forecast disturbance error. By developing expression (8) a little more, we find

$$X_t - X_{t-1} = -\frac{\lambda}{g}(d_t - \hat{d}_t). \tag{9}$$

Comparing expression (1) and writing expression (8) as $X_t = X_{t-1} + \frac{\lambda}{g} e_t$, it can be said that control proportional to the error was established, in the discrete case. This proportionality is the amount that measures the difference between the value that the variable should present and its current value.

Since the sample data are collected and measured in even intervals, and the adjustments shall also be done in even intervals in relation to the disturbance of each variable, this is considered to be a discrete adjustment system in relation to the disturbance of each variable. In this manner, Eq. (9) provides the adjustment level that must be carried out in regards to the compensation variable. Constant g represents the system gain, which measures the system quality changes that occurred in the variable (X_t) for each unit altered, represented by the largest coefficient of the regression equation. This is estimated through the VAR (vector auto-regression) model, capturing the joint effect of the variables. The smooth constant λ shall be the one that provides the lowest forecast error of the series of errors of the adjusted disturbances provided by EWMA statistics.

According to Montgomery and Mastrangelo (1991), when the temperature is controlled by adjusting the positioning of a control dial, EWMA statistics may be applied to the adjustment series of the control dial, or equivalently, to the output sign that directs the positioning of the control dial. And if the adjustment algorithm is working properly, problems that affect temperature shall be reflected in the dial adjustments. We noted that EWMA statistics in the period t is the same as EWMA in the period $t - 1$

plus a λ fraction of the error forecast one step ahead, and in this manner it is easy to see that EWMA is simply the proportional term to the error (Hunter, 1986).

The idea of working with the proportional adjustment to the last error derives from the fact that it is the forecast of the disturbance and this adjustment shall occur in each stage to cancel such forecast, using the smooth constant λ applied to the most recent observation, that is, to the last error, which also avoids that an excessive compensation be applied in the system (Box & Luceño, 1994, 1997; Montgomery, Keats, Runger, & Messina, 1994). Since the disturbances are forecast and then canceled at each stage, there is no need for performing an integral control, which would represent the sum of all the past residuals, which shall not influence the system, since in addition to already being corrected, we would not know the real disturbances that would affect the system.

The control variable in an autoregressive process will be the own series in study in the prior instant (Del Castillo, 1996), because when the production system is in lots, the observations form a series, making it possible to forecast the values of the variables to be adjusted, and in turn, these values are used to adjust the current lot.

3. Implementation and interpretation of the proposed controller

For an adjustment to succeed, two tasks are required: an identification process and the appropriate application of the adjustment rules. With assistance from univariated and multivariate control charts, the stability of the productive system was verified. The variables shall be identified by using the analysis of the main components and the correlation analysis carried out between the main components and the original variables. For calculating the correlation, we shall use the components that present the largest instability, classified through the control charts.

After identifying the variables that will be adjusted, one should first find the target values for each production lot. This value shall be generically represented by the letter A. The objective is to maintain the process the closest possible to this value A, which shall be done through the manipulation of the input variables.

The result in expression (9) supplies the adjustment level that should be applied to the compensation variable, and its sign will indicate the number of units that the control dial should be rotated, since the dial is equipped with a measurement scale to facilitate adjustment. If (+), the regulator dial should be rotated clockwise, liberating energy; if (−), the dial should be rotated counterclockwise, reducing the amount of energy. In the multivariate system, more than one variable should undergo feedback adjustment, and it should be pointed out that each variable will have its own value, possibly of varying magnitudes.

The constant g represents the system gain, measuring the incurred alterations of the variable in question of the system quality for each unit altered in the variable (X_t), represented by the largest coefficient of the regression equation. The variable is estimated simultaneously using the seemingly unrelated regression (SUR) approach, so that the correlation between the stochastic terms is used to improve the quality of the estimates, capturing the inter-relationship among the variables and the errors (Enders, 1995; Hill, Griffiths, & Judg, 1999). This class of SUR models belongs to the VAR class, proposed by Zellner (1962).

The variable control in an autoregressive process will be the series studied in the previous instant, Del Castillo (1996). When the production systems are in lots, the observations form a series, making it possible to adjust the values of the variables using the forecasted value; next, the forecasted value of the lot or the previous time in the series will be used to adjust the current lot.

The smooth constant λ, will be found using EWMA statistics (Roberts, 1959), since according to Box and Jenkins (1976), this statistical method is able to minimize the output variable through adjustments to input variables in the process. Continuous measurements should be carried out to evaluate how far the output variable is from the established target.

According to Montgomery and Mastrangelo (1991), when the temperature is controlled by adjustment using a dial or a valve, EWMA statistics can be applied to the series of valve adjustments and the controller's sign will direct the dial or valve positioning. If the adjustment algorithm is working properly, problems that affect the temperature will be reflected in the valve adjustments. One should note that EWMA statistics in the period t is the same as EWMA in the period $t-1$ plus a fraction of the error's smooth constant λ forecasted one-step-ahead (Hunter, 1986). Hence, EWMA is just the term proportional to the last error. Using this methodology, we tried to establish a discerning routine to calibrate the variables with the objective of maintaining the output variables as close as possible to the established target.

4. Multivariate autoregressive models

Researchers and workers in the industrial field frequently obtain data that present several answers for a given process, and this set of variables should be controlled. When the answer structure is multivariate, a problem emerges regarding estimation of the parameters that do not exist in the univariated case, for the parameter vector to be estimated must take into consideration the interrelationships between the variables (Khuri & Conlon, 1981). The methodology of regression by Autoregressive Vectors (*VAR*) enables a joint estimation of the parameters, whereby the interrelationships are considered and the dynamic behavior of the data is captured. This provides an idea of the structure of the relationship between the system's input and output variables.

VAR enables the analysis not only of the individual behavior of each series, but also of the possible relationships between the series and the dynamic relationships that occur between them in a given time period. In this manner, it is possible to increase the accuracy of the estimates of the model, using the additional information provided by the interrelationships, providing a reliable measurement with which to carry out the feedback adjustment. The models, both univariate and multivariate, are much discussed by authors such as Box and Jenkins (1970), Charenza and Deadman (1997), Hamilton (1994), Lütkepohl (1991), Maddala (1992), and Reinsel (1993) who present the multivariate case as a generalization of the univariate one.

An autoregressive vector is simply a system of dynamic linear equations in which each variable is written as a function of a serially non-correlated variable and all the variables that belong to the system have the same number of lags, represented by p. These lags determine the order of the model, which is generically represented by *VAR(p)*, as can be seen in Eq. (10)

$$\mathbf{Z}_1 = \boldsymbol{\nu} + \boldsymbol{\varphi}_1 \mathbf{Z}_{t-1} +, \ldots, + \boldsymbol{\varphi}_p \mathbf{Z}_{t-p} + \boldsymbol{\varepsilon}_t \tag{10}$$

In Eq. (10), it is held that $\mathbf{Z}_t$ is a random vector, $\boldsymbol{\varphi}_i$ is the matrix of the coefficients, $\boldsymbol{\nu}$ is the vector of the intercepts, allowing the process average to be different from zero, and $\boldsymbol{\varepsilon}_t$ is the white noise vector, also termed process innovation vector, that is: $E(\varepsilon_t) = 0$ *e* $E(\varepsilon_t \varepsilon_t') = \boldsymbol{\Sigma}$, where $\boldsymbol{\Sigma}$ is the matrix of non-singular variance-covariance $E(\varepsilon_t \varepsilon_s') = 0$ for $s \neq t$.

In the autoregressive model of first order, *VAR(1),* described as follows

$$\mathbf{Z}_t = \boldsymbol{\nu} + \boldsymbol{\varphi}_1 \mathbf{Z}_{t-1} + \varepsilon_t, \tag{11}$$

taking the time factor, $t = 1, 2, \ldots, t$, one can write the following equations

$$\mathbf{Z}_1 = \boldsymbol{\nu} + \boldsymbol{\varphi}_1 \mathbf{Z}_0 + \varepsilon_1 \tag{12}$$

$$\mathbf{Z}_2 = \boldsymbol{\nu} + \boldsymbol{\varphi}_1 \mathbf{Z}_1 + \varepsilon_2, \tag{13}$$

substituting Eq. (11) in Eq. (12), one obtains the equation

$$\mathbf{Z}_2 = \boldsymbol{\nu} + \boldsymbol{\varphi}_1(\boldsymbol{\nu} + \boldsymbol{\varphi}_1 \mathbf{Z}_0 + \varepsilon_1) + \varepsilon_2 = (I_k + \boldsymbol{\varphi}_1)\boldsymbol{\nu} + \boldsymbol{\varphi}_1^2 \mathbf{Z}_0 + \boldsymbol{\varphi}_1 \varepsilon_1 + \varepsilon_2 \tag{14}$$

$$\vdots$$

$$\mathbf{Z}_t = (I_k + \boldsymbol{\varphi}_1 + \cdots + \boldsymbol{\varphi}_1^{t-1})\upsilon + \boldsymbol{\varphi}_1^t \mathbf{Z}_0 + \sum_{i=0}^{t-1} \boldsymbol{\varphi}_1^i \varepsilon_{t-i}. \tag{15}$$

Observing Eqs. (14) and (15), one notes that the autoregressive process is determined by an initial value followed by the previous random shocks. The vectors $\mathbf{Z}_1, \ldots, \mathbf{Z}_t$ are determined solely by $\mathbf{Z}_0$, which is the initial value plus the sum of $\varepsilon' s$.

In this manner, it is clear that the multivariate model (15) can be represented by an infinite sum of lags of errors plus the value of $\boldsymbol{\varphi}_1^t \mathbf{Z}_0$, which will tend towards zero when t tends towards the infinitive. Thus, the infinite autoregressive vector can be better termed as the finite moving average vector. The *VAR(1)* model, whose autovalues $\boldsymbol{\varphi}_1$ are less than 1, shall have stable parameters, which is a condition that must be satisfied in order for the model to provide good forecasts (Cochrane, 1997).

The multivariate system must present a white noise process so that the errors are independent and identically distributed, that is, $\varepsilon_t \approx \text{iid}N(0, \Sigma)$. This condition guarantees the absence of any serial correlation in the errors, in other words, the residuals are homoscedastic, presenting a constant variance.

According to Charenza and Deadman (1997), one can observe that the terms of the errors of a multivariate autoregressive model of first order are contemporaneously correlated, that is, $E(\varepsilon_{1t}) = E(\varepsilon_{2t}) = 0$; $E(\varepsilon_{1t}^2) = \sigma_{11}$; $E(\varepsilon_{2t}^2) = \sigma_{22}$; $E(\varepsilon_{1t}\varepsilon_{2t}) = \sigma_{12}$.

In order to obtain the uncorrelated errors, a weight by means of the variances must be carried out to obtain the non-correlated errors. The interrelationship between the errors is neutralized by the weight of the variables through the variances and covariances of the errors.

According to Enders (1995), if some of the equations have regressors that are not included in the others, different variables on the right side of each equation or even if the variables have different lag times, one must use the SUR estimator, in order to obtain efficiency in the VAR coefficient estimates, obtaining a model termed 'near VAR.' The manner in which these parameters are estimated shall be shown next.

On the conditions of simultaneity described previously, Zellner (1962) demonstrates that the method of seemingly uncorrelated regressions (SUR) enables an estimation that is asymptotically more efficient than one estimated equation by equation. An assumption that enables the use of a combined estimation process—which is better than using the least squares method separately—is the connection of equations through errors. This assumption states that the stochastic terms in the equations, in the same instant, are correlated. The addition of the contemporaneous correlation assumption effectively introduces additional information not included when the least squares estimation is done separately (Hill et al., 1999). The use of *SUR* provides information on the correlation between the stochastic terms, and for this reason is more precise than the least squares process, and this fact is corroborated by the lower standard deviations of the estimates.

The *SUR* estimation method allows for each equation to have its own functional form, taking into consideration only the existing correlation between the errors of the Zellner equations (1962).

4.1. An example in a tile plant

During tile production, the only stages that do not allow for flexibility are the burning and drying stages. Therefore, once it begins, the burning process cannot be interrupted or reversed, and one must wait until the ceramic piece completes the burning cycle. The oven cannot be turned off frequently, because since the time for cooling and calibration is around 27 days, the delay would represent a great damage. The oven is composed of three heating zones. The first is termed pre-heating and has average temperatures around 400 °C, represented by the variables PA1, PA2, PA3; the second is termed zone of heating or burn zone with temperatures that vary from 600 to 1100 °C, represented by the variables AQ1, AQ2, AQ3, AQ4, AQ5, AQ6; and, finally the cooling zone with temperatures around 600 °C represented by the variables RF1, RF2 and RF3. These variables are arranged sequentially along 87 m of a tunnel oven.

Multivariate feedback will be necessary because a group of variables will be analyzed simultaneously, taking into consideration their inter-relationship. These variables—a total of 12—are the temperatures of the burn points of the oven, forming a series containing 92 observations each, taken in intervals of 1 h in the three burn zones.

Ceramic materials depend on the time they remain in each burn stage, on pre-heating time, burn time, cooling time, and mainly on the uniformity of temperature in each stage. The biggest problems are different gauges and sizes, low resistance materials, cracks, and materials with different coloration, all which lead to lower prices and hence lower profit for the producer.

The modeling of the selected variables shall be carried out by the methodology proposed by Zellner (1962). This way, it will be possible to determine the disturbances that each variable shall present.

The steps to follow for this procedure are as follows:

- Estimate the equations separately using least squares;
- Use the residuals of least squares of the previous step to estimate the variances and covariance of the errors;
- Use the estimates of the error variances to estimate the equations together;

As we wish to capture the inter-relationships between the variables, we use the variable previous to the one identified as out of control and the next variable, because we are working with variables that are sequentially distributed, then a set of three variables is used. The principal components analysis was used to reduce the number of variables to be used, but mainly led to the identification of the variable or set of variables that would be the possible cause of instability in the system.

We used a group formed by three variables—distributed sequentially—to be modeled because it is identified as being responsible for the stability of the temperatures in the heating and cooling stages. Since these variables are interrelated and their errors are correlated, the NEAR VAR methodology will be used to achieve a good level of efficiency in the estimates (Enders, 1995; Zellner & Theil, 1962). The grouped variables will be considered dependent, and the system's other variables will enter as independent ones, forming in this manner the equation system to be estimated.

The first group of variables to be modeled is formed by AQ6, RF1, RF2, and the second group to be modeled is formed by AQ1, AQ2 and AQ3. The system's other variables will be considered with a lag of four periods. In this manner, not only the adjustment level for the identified variable is obtained, but also the adjustment level that should be applied to the group of variables. Tables 1 and 2 present the adjusted values for the first and second group of identified variables, which were estimated using PcFiml and PcGive software.

According to Sachs et al. (1995)) the so-called 'Run-by-Run' control structure is not restricted to a model of first order and could be used for superior orders, as long as the control function maintains process stability. Table 3 shows the variables with their target values, the forecasted value and the disturbance for each one of the series in study.

Until this point, we have determined each variable's disturbance, but it is still necessary to determine the level of adjustment that should be applied to each selected variable, through the oven's temperature control dial.

Table 1
Estimation of the first set of variables identified by the variables AQ6, RF1 E RF2

Variable	Coefficient	Standard error
Modeling of variable $(AQ6)_t$		
Constant	318.19	123.11
$(AQ6)_{t-1}$	0.59543	0.080679
$(RF2)_{t-1}$	−0.20977	0.081842
$(AQ5)_t$	0.20646	0.052096
Modeling of variable $(RF1)_t$		
Constant	298.26	119.55
$(RF1)_{t-1}$	0.78693	0.051227
$(AQ2)_{t-2}$	−0.27603	0.097528
$(AQ5)_{t-1}$	0.10266	0.047431
$(PA2)_{t-1}$	0.13931	0.054475
$(PA1)_{t-2}$	−0.13698	0.060355
Modeling of variable $(RF2)_t$		
Constant	417.77	82.171
$(RF1)_{t-2}$	0.24361	0.049103
$(RF2)_{t-1}$	0.63258	0.069200
$(AQ1)_t$	−0.16966	0.051740
$(AQ2)_{t-2}$	−0.16736	0.063493
$(AQ5)_{t-2}$	−0.090479	0.032751

Table 2
Estimation of the second set of variables identified by the variables AQ1, AQ2 E AQ3

Variable	Coefficient	Standard error
Modeling of variable $(AQ1)_t$		
Constant	373.33	76.619
$(AQ1)_{t-1}$	0.37055	0.091903
$(RF1)_{t-2}$	0.15948	0.061547
Modeling of variable $(AQ2)_t$		
Constant	735.69	129.01
$(AQ1)_{t-2}$	0.15657	0.075496
$(AQ2)_{t-1}$	0.33724	0.093757
$(AQ3)_{t-2}$	−0.24767	0.087791
Modeling of variable $(AQ3)_t$		
Constant	935.66	99.707
$(AQ5)_t$	0.25924	0.050534
$(AQ6)_{t-2}$	−0.17007	0.0725519

4.2. Determining the smooth constant λ

To apply the feedback equation, it is still necessary to know the value of the smooth constant λ, shown next.

The selection method of constant λ is provided by Crowder (1987) and Lucas and Saccucci (1990), whereby the one that presents the best performance for the *EWMA* chart in terms of *ARL* is selected. Considering the correlated data, Montgomery and Mastrangelo (1991) suggest selecting the value of λ based on the minimization of the square sum of errors.

The value of the smooth constant λ will be the one to offer the smallest forecast error of the series of disturbance errors adjusted by EWMA statistics, as suggested by Montgomery and Mastrangelo (1991). To find the smooth constant, a search was carried out in the interval [0.1; 0.99] with increments of 0.01. This determined the best value for λ that presented the minimum square error, which was 0.1 for all variables, that is showed in Table 4.

Table 3
Target values, forecasted values and disturbance of the variables that will undergo feedback control

Variables	Target value	Forecasted value	Disturbance
AQ6	1030.87	1034.661	−3.791
RF1	806.37	802.5887	3.7813
RF2	607.859	604.7431	3.1159
AQ1	797.989	797.9714	0.0176
AQ2	935.326	935.4515	−0.1255
AQ3	1045.62	1045.643	−0.023

Table 4
Values of λ that provided the lowest sum of the squares of errors

Variables	Values of λ	Sum of the squares of errors
AQ6	0.1	498.9784
RF1	0.1	322.3064
RF2	0.1	350.3542
AQ1	0.1	498.9784
AQ2	0.1	322.3064
AQ3	0.1	350.3542

4.3. Application of the control equation for system feedback

In this section we will determine the adjustment level that should be introduced in the system to maintain the temperature as close as possible to the target value. The system is inspected and regulated in one-hour intervals, since the alteration introduced in the system will be applied in the next instant, that is, within the period of 1 h. This is termed responsive system (Box & Luceño, 1997).

The concept of controllers it is to separate the problem of estimation from the control problem (Astrom & Wittenmark, 1989). It is interesting that an algorithm of recursive estimation supplies the estimate parameters for the controller's composition (Del Castillo, 1996). To apply the control equation developed in item 2, it is sufficient to know the disturbance values of each variable, the constant g value that will determine the effect in the productive system, and the smooth constant value:

Control equation for the first group	Control equation for the second group
$(\mathrm{AQ6})_t - (\mathrm{AQ6})_{t-1} = -(0.10/0.59543)$	$(\mathrm{AQ1})_t - (\mathrm{AQ1})_{t-1} = -(0.1/0.37055)$
$(-3.791) = 0.63668$	$(0.0176) = -0.00475$
$(\mathrm{RF1})_t - (\mathrm{RF1})_{t-1} = -(0.1/0.78693)$	$(\mathrm{AQ2})_t - (\mathrm{AQ2})_{t-1} = -(0.1/0.33724)$
$(3.7813) = -0.48051$	$(-0.1255) = -0.03721$
$(\mathrm{RF2})_t - (\mathrm{RF2})_{t-1} = -(0.1/0.63285)$	$(\mathrm{AQ3})_t - (\mathrm{AQ3})_{t-1} = -(0.1/0.25924)$
$(3.1159) = -0.49236$	$(-0.023) = 0.00887$

Through these equations, it is possible to know the adjustment level that should be applied to each variable to maintain the process as close as possible to target. The controlling dial is equipped with a scale, making it possible to apply an adjustment to variable AQ6 by rotating the dial by 0.63668, that is, by 0.6 units clockwise, rotating 0.5 units. The temperature control dial of variable RF1 counterclockwise and, finally, to apply an adjustment of 0.5 units to variable RF2, rotating its control dial counterclockwise. In the second point one should note that the disturbance found in these variables is very small, thus in this case a feedback adjustment is unnecessary. Still, a monitoring process is necessary.

It should be pointed out that, in this case, as the process is correlated to the modification or the alteration in one of the variables, it can alter the other variables. Therefore, in this research, the proposal is to evaluate and accomplish feedback adjustment for groups of variables.

5. Conclusion

The objective of this study was to develop an auxiliary methodology for the monitoring and/or feedback of a multivariate system, clearly stating all the steps to be followed and facilitating the application of the method, while also connecting statistical control with EPC.

In the composition of the proposed controller, the smooth λ and system gain g constants were found in an efficient manner, rendering the adjustment efficient, since, according to Sachs et al. (1995), feedback compensations are frequently taken based on the previous experience of professionals connected to the process. The authors believe that the adjustment process represents a significant advance in the process control technology, which recognizes the nature of the data and provides a structure with which to control it.

To obtain the proposed controller, after the identification of the variables to be feedback, distinct phases must be completed until the final execution. These phases are: modeling, identification, estimation, the control project and monitoring. For this reason, the success of the application of algorithmic statistical process control (ASPC) requires the cooperation not only of the professional in charge of making the adjustments, but of professionals who are knowledgeable specifically in quality control and temporal series, in order to ensure the successful performance of the proposed methodology (Tucker, Flatin, & Wiel, 1993). This knowledge is important for the proper elaboration of models that represent the process, to determine the experimental conditions, estimate parameters, validate results, determine the appropriate control, implement and monitor the procedure. In many practical applications, technological support and knowledge are not always available for elaborate methods, but there is a need to promote an advance in this area since the most difficult problems require specialized knowledge to render the company more competitive. However, the efforts of all production professionals and management support are fundamental for the implementation of this methodology. The application of *ASPC* through the proposed methodology is a very useful tool in the industrial field, for it allows for the adjustment of the productive process to be made while the parts or products are still on the production line.

Yet at the same time that it is useful, there is also the risk of improper use of the methodology due to lack of knowledge on behalf of professionals. For this reason we suggest training for the people involved in the productive process, so that the proposed methodology may be adopted with success.

For a wider use of this research it would be interesting to apply it to other types of burning ovens that have the same characteristics of the furnaces for burning ceramic pieces. It would also be useful to apply it to multivariate processes that do not present the characteristics of variables in a linear sequence, in processes such as beverage bottling, for example, which present characteristics such as pressure, quantity of gas to be injected into the container, and volume of the liquid to be bottled, all of which must be continuously controlled.

By applying methodology to a real case, it was possible to locate the out-of-control variables, with the advantage of providing the operator with a starting point for system monitoring and/or feedback control, to apply the necessary changes to the correct variables, without having to randomly choose which variables should be adjusted.

The determination of the constant λ and the gain of the system g in the feedback equation show how multivariate feedback can be reached based on scientific grounds. The group estimation of the set of variables was able to capture the inter-relationship between them, determining in this manner the effect of each variable in the system.

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